MAT 140: Differential Calculus with Review

Midterm Exam 2: The "Ain't No Mountain High Enough" edition

Form B - Date: 10/17/2012

Instructions:

1. ***Show your work for all calculations you do.***
(Correct answers WITHOUT work shown may receive only partial credit.
Incorrect answers WITH work shown may also receive partial credit.)

2. Time management: use your time wisely. Don't get hung up.
Make sure you solve all the easy ones.

3. Keep your eyes on your own test.

4. If you finish early, CHECK YOUR WORK.

Name: _____

Table below used for grading purposes:
(Do not attempt to fill it in yourself. I am not so easily fooled...)

Problem #	Possible Points	Missed Points
1	16	
2	12	
3	10	
4	15	
5	12	
6	10	
7	6	
8	10	
9	6	
10	3	
∞	(+2 E.C. bonus)	
	Total missed:	
TOTAL Score	**100**	

Problem 1: This little triggy went to market

(2 points) Convert $\frac{\pi}{20}$ radians to degrees: _____

(2 points) Convert 70° to radians: _____

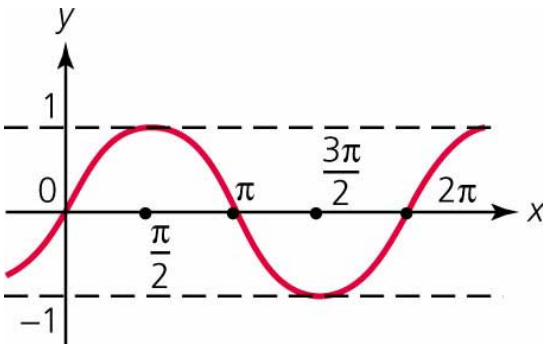
(2 points) *(Give EXACT answer)* $\sin\frac{7\pi}{6} =$ _____

(2 points) *(Give EXACT answer)* $\tan\frac{7\pi}{6} =$ _____

(2 points) *(Give EXACT answer)* $\cos 111\pi =$ _____

(4 points) Find ALL values of θ that satisfy the equation $\csc\theta = \sqrt{2}$

(2 points) The graph shown at right represents which function:



a) $y = \sin\left(x + \frac{\pi}{2}\right)$ b) $y = \cos\left(x - \frac{\pi}{2}\right)$

c) $y = \sin\left(x - \frac{\pi}{2}\right)$ d) $y = \cos\left(x + \frac{\pi}{2}\right)$

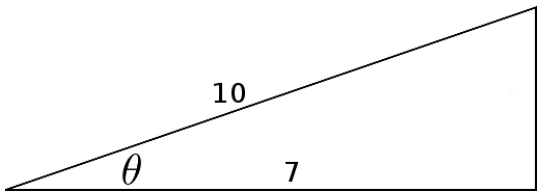
e) $y = \tan x$ f) $y = \sec x$

g) $y = \csc x$ h) $y = \cot x$

Problem 2: This little triggy stayed home

(4 points) Recall that the graph $y = \sin x$ has a *period* of 2π, because the wave shape begins to repeat itself exactly after a length of 2π units. Here is another way of thinking about it: the period (2π) is the shortest non-zero distance you can shift the function so that the graph will look exactly the same as before. The graph of $y = \cos x$ also has a period of 2π. Consider the graph $g(x) = 10 + 3\sin\frac{x}{2}$. What is the *period* for this graph?

(4 point) Given the right triangle shown over there ------→
what is the value of $\sin\theta$?
Express answer as a decimal rounded to the nearest hundredth.



(4 points) A ladder 50 feet long is leaned up against a vertical wall, with the angle at the base of the ladder being 80 degrees. How high up the wall does the ladder reach? *(Round answer to the nearest hundredth.)*

Problem 3: This little triggy had an identity crisis

(5 point) Use trig identities to show that $\dfrac{\cos 2x}{\cos x + \sin x} + \sin x = \cos x$

(5 point) Use trig identities to show that $\dfrac{\sin^3 x}{\cos x} + \cos^2 x \tan x - \tan x = 0$

Problem 4: This little triggy took it to the limit.

(5 points) SHOW WORK! Find $\lim_{x \to \frac{\pi}{4}} \frac{\tan x}{x} =$

(5 points) SHOW WORK! Find $\lim_{x \to 0} \frac{\sin 3x}{2x} =$

(5 points) SHOW WORK! Find $\lim_{x \to -\infty} \frac{6x^5 - 7x}{2x^3 + x^2} =$

Problem 5: That slippery slope, and its slippery limit.

(2 points) The slope of the line PERPENDICULAR to a function f(x) at x = a is given by:

a) $f'(-a)$ b) $f'(a)$ c) $\dfrac{-1}{f'(a)}$ d) $f(a)$ e) $f(-a)$ f) $\dfrac{-1}{f(a)}$

(6 points) Use a limit definition of derivative to find $f'(2)$ given that $f(x) = 2x^2 + 1$.
(You must show your work and you must use one of the two limit definitions, NO shortcuts.)

(4 points) Use the previous answer to find an equation for the line tangent to $f(x)$ (given above) at $x = 2$.
Express your final answer in the form $y = mx + b$.

Problem 6: Vertigo, vertical, vertex.

(6 points) Given that $f(x) = 3x^2 - 30x + 100$, find $f'(x)$.

(4 points) Use calculus to find the vertex of the parabola given by $f(x) = 3x^2 - 30x + 100$.

Problem 7: SEE KANT. SEA CANT. C CAN'T. SECANT.

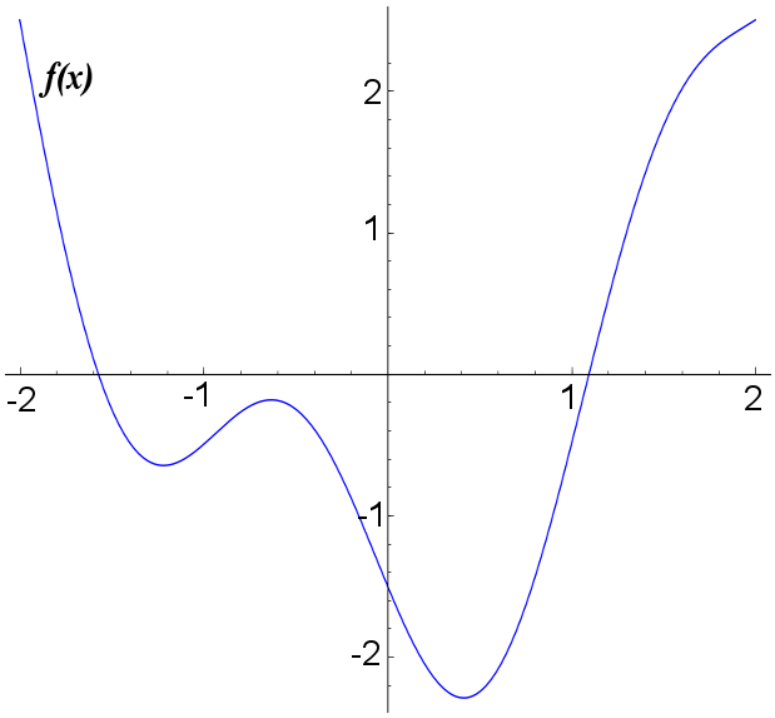
(6 points) Find the slope of the SECANT line to the function $f(x) = \sqrt{x}$ that intersects the function at the x-values $x = 1$ and $x = 4$.

Problem 8: More different. More differentiation.

(4 points) Given that $g(x) = x^{10} - \sqrt{x} + \dfrac{4}{x^2}$, find $g'(x)$.

(6 points) Given the graph of $f(x)$ at right, fill in the blanks, using each of the following answer choices AT MOST once. (There is one unused extra choice in the list.)

-10, −3.1, −1.5, 0.0, 1.1, 2.0, 5.1



$f'(-1) =$ _____

$f'(0) =$ _____

$f'(0.415) =$ _____

$f'(1) =$ _____

$f(0) =$ _____

$f(1.6) =$ _____

Problem 9: Of intermediate difficulty

(6 points) Use the Intermediate Value Theorem to show that the function $y = 1 + |x| - x^2$ has at least two x-intercepts on the interval [-3,3]. Sketch a graph, and CLEARLY explain your argument/reasoning in English.

Problem 10: Leftovers

(3 points) Would you like 3 points for free (so that the possible points sum to 100)? Yes / No

Problem ∞: Early explorers of Mt. Calculus

(+2 E.C. bonus points) The "prime" notation for derivatives, such as *y'* and *f'(x)*, was introduced by French mathematician Joseph Louis Lagrange. Who introduced the alternative notation $\frac{d}{dx}$?